Solutions

7.3: Probability and Probability Models

Definition 1. A (finite) probability distribution is an assignment of a number $P(s_i)$, the probability of s_i, to each outcome of a finite sample space $S = \{s_1, s_2, \ldots, s_n\}$. The probabilities must satisfy

1. $0 \leq P(s_i) \leq 1$ and
2. $P(s_1) + P(s_2) + \cdots + P(s_n) = 1$.

Definition 2. A probability model for a particular experiment is a probability distribution that predicts the relative frequency of each outcome if the experiment is performed a large number of times.

Example 1. (Uniform Distributions) A total of 1.9 million hybrid vehicles had been sold in the US through October of 2011. Of these, 955,000 were Toyota Prii, 205,000 were Honda Civics, 170,000 were Toyota Camrys, 105,000 were Ford Escapes, and the rest were other makes.

(a) What is the probability that a randomly selected hybrid vehicle sold in the US was either a Toyota Prius or a Honda Civic?

(b) What is the probability that a randomly selected hybrid vehicle sold in the US was not a Toyota Camry?

randomly selected = uniform distribution

(a) $P(\text{Prius} \cup \text{Civic}) = \frac{955{,}000 + 205{,}000}{1.9 \text{ million}}$ (b) $P(\text{not Camry}) = 1 - P(\text{Camry}) = 1 - \frac{170{,}000}{1.9 \text{ million}}$

Example 2. Suppose you are trying to scam a casino, so you bring a weighted die with you. This die is weighted such that 6 is three times as likely to come up as any one of the other numbers. Find a probability distribution for your weighted die and calculate the probability of rolling an even number.

x = prob. of a number besides 6

y = prob. of rolling 6.

Then $y = 3x$ and $y + 5x = 1$.

So $1 = y + 5x = 3x + 5x = 8x$ and $x = \frac{1}{8} = 0.125$.

1	2	3	4	5	6
1/8	1/8	1/8	1/8	1/8	3/8

$P(\text{even number}) = \frac{1}{8} + \frac{1}{8} + \frac{3}{8} = \frac{5}{8}$.

Unions, Intersections and Complements From our knowledge of cardinality of sets and condition 2 above,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \text{and} \quad P(A') = 1 - P(A).$$